We explore the structure and thermodynamics of water clusters confined in nonpolar cavities. By calculating the grand-canonical partition function term by term, we show that small nonpolar cavities can be filled at equilibrium with highly structured water clusters. The structural and thermodynamic properties of these encapsulated water clusters are similar to those observed experimentally in the gas phase. Water filling is highly sensitive to the size of the cavity and the strength of the interactions with the cavity wall. Water penetration into pores can thus be modulated by small changes in the polarity and structure of the cavity. Implications on water penetration into proteins are discussed.

**Methods**

The water occupancy of a cavity is determined by the balance of the excess chemical potentials of water inside the cavity and in the bulk fluid outside. We restrict our analysis to spherical cavities with smooth walls and rigid fullerenes to elucidate the fundamental aspects of filling thermodynamics and structure. In the grand-canonical ensemble, water molecules inside a cavity are in thermal and chemical equilibrium with the surrounding bulk fluid. The occupancy probabilities $P(N)$ for finding exactly $N$ particles in the volume $V$ are related through (28)

$$
\frac{P(N+1)}{P(N)} = \frac{\rho V e^{\beta \mu_{\text{bulk}}}}{N+1} \langle e^{-\beta (\mu_{N+1} - \mu_{N})} \rangle_N,
$$

where $\rho$ is the number density of the bulk phase, $\mu_{\text{bulk}}$ is the corresponding excess chemical potential, $\beta = 1/k_B T$ with $k_B$ being Boltzmann’s constant and $T$ being the absolute temperature, $\mu_N$ is the potential energy of the $N$-particle system, and $\langle e^{-\beta (\mu_{N+1} - \mu_{N})} \rangle_N$ is a canonical test-particle average for an $(N+1)$th water molecule randomly inserted into the cavity containing $N$ water molecules. This expression allows us to calculate the occupancy probabilities $P(N)$ successively, starting from $N = 1$, by a combination of test-particle insertion and the method of overlapping histograms (28, 29). The probability of finding an empty cavity, $P(0)$, is obtained by normalization. The free energy $\Delta A_N$ of filling the cavity with exactly $N$ water molecules from bulk phase is related to the occupancy probabilities $P(N)$ by $\Delta A_N = A_N - A_0 = -k_B T \ln P(N)/P(0)$, neglecting a pressure-volume term that is small near ambient conditions (28). From the temperature dependence of $\Delta A_N = \Delta U_N - T \Delta S_N$, we can estimate the internal energy $\Delta U_N$ and entropy $\Delta S_N = -\Delta A_N / \partial T$ of the water clusters inside the cavities relative to bulk water.

The occupancy probabilities $P(N)$ were evaluated by performing Monte Carlo simulations of water molecules in the canonical ensemble (28) for four different spherical cavities with diameters of 0.9, 1.0, 1.04, and 1.19 nm at temperatures of $T = 280, 290, 298, 310$, and $320$ K. The free volume of the cavities, defined by a 1 $k_B T$ energy surface for the water-cavity interaction, ranges from 0.02 to 0.13 nm$^3$ for diameters between $2R = 0.9$ and 1.19 nm.

Data were collected over half a million Monte Carlo steps after an initial equilibration of 100,000 steps. Additional simulations were performed for water inside rigid fullerenes $C_{140}$ and $C_{180}$ with diameters of $\approx 1.04$ and $\approx 1.19$ nm, respectively. We are not concerned here with the actual mechanism of how water gets as the specific heat, to clarify the factors that determine the formation and stability of the water clusters. The 3D structures of the more stable clusters in the cavities are examined and compared with water clusters studied experimentally in the gas phase (26).

**Water clusters in nonpolar cavities**

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into or out of the cavities. As in our earlier studies of carbon nanotubes (12, 13), we used the three-point transferable intermolecular potential function (TIP3P) model (30) to describe water, and the sp$^2$-graphene" carbon type of AMBER (parm94) (31) with Lennard–Jones parameters for the carbon–water interaction. For water in the bulk phase, we use the TIP3P model (30, 32). The experimental excess chemical potential of TIP3P water at ambient conditions, $\mu_{\text{TIP3P}}^N(298) = -25.3 \text{ kJ/mol}$ (12, 28).

### Results

#### Thermodynamics of Cavity Filling

Fig. 1 summarizes the thermodynamics of water inside the nonpolar cavities. The free energy profiles as a function of the number $N$ of water molecules in the cavities reflect two-state behavior with minima corresponding to empty ($N = 0$) and filled ($N > 0$) states. As the size of the cavity increases, the filled state becomes more populated and its free energy decreases. For the graphene potential (Fig. 1A), the filled state is preferred over the empty states for cavities of 1.0 and 1.04 nm in diameter that contain clusters of three and four water molecules, respectively. Reducing the van der Waals attraction between water and the cavity walls makes filling unfavorable at all cavity sizes considered (Fig. 1B). For both types of cavity–water interactions, $C_{140}$ and $C_{180}$ fullerenes give results similar to those of smooth spherical cavities of corresponding size.

To identify the thermodynamic driving forces of the filling process, we calculate the energy and entropy of the system as a function of the number of water molecules inside the cavity. The value of $\mu_{\text{TIP3P}}^N(298)$ is shifted to the

\[
\mu_{\text{bulk}}(T) = \mu_{\text{TIP3P}} + 0.05136(T - 298 \text{ K}) \text{kJ/(mol K)} - 6.678 \times 10^{-3}(T - 298 \text{ K})^2 \text{kJ/(mol K}^2) \]

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turbation theory with respect to the water–wall van der Waals interaction energy (Fig. 1C). In contrast to the energy, the entropy of transfer is insensitive to changes in the water–cavity potential (Fig. 1D). As the occupancy $N$ rises, the entropy per molecule decreases, reflecting the formation of hydrogen-bonded water structures and a reduction in free volume.

**Structures of Encapsulated Water Clusters.** The structures of the encapsulated water clusters help us to understand their stability and highlight the relationship to experimentally studied water clusters in the gas phase (26). Fig. 2 shows clusters of $N = 3$ to eight water molecules inside the smooth cavities and a nine-water cluster corresponding to the filling free energy minimum of the C$_{180}$ fullerene. For Fig. 2, low-energy structures were selected by tracing the energy landscape of equilibrated clusters during the Monte Carlo simulations.

The structures of the water clusters formed inside the large nonpolar cavity are strikingly similar to the structures deduced from gas-phase spectroscopy (26). The water clusters are all hydrogen-bonded, beginning with the dimer, and evolve through the trimer to the pentamer as cyclic structures with each water monomer donating a hydrogen bond to one nearest neighbor and accepting a hydrogen bond from the other. In gas-phase experiments, the free hydrogens of each monomer in the ring lie alternately above and below the plane of the oxygen atoms, thus creating a frustrated structure for the trimer. In our simulations using the TIP3P model for water, the free hydrogens are nearly planar and are perfectly planar at a $T = 0$ K structure. In simulations using the anisotropic site–site potential (ASPW4) (33) which is a refinement of the Millot–Stone potential (34, 35) with superior performance for the water tetramer in C$_{140}$ (36), a plot of the specific heat versus temperature indicates a “melting” transition near 180 K of the water octamer encapsulated into C$_{180}$ (Fig. 3). The corresponding transition occurs near 400 K for the water tetramer in C$_{140}$. The nearly cubic hydrogen-bonded octamer is transformed into a mostly disordered liquid with broken hydrogen bonds above the “melting” temperature of 180 K, with only infrequent occurrence of the ordered clusters. Such melting has been studied previously (36–38). The melting of the tetramer, which occurs at a higher temperature of 400 K, is likewise characterized by the breakup of its ring structure. To our knowledge this has not been observed in the gas phase.

**Concluding Remarks**

Our studies show that water molecules can fill even small nonpolar cavities at equilibrium and ambient conditions. The smallest stable water cluster in a spherical cavity (1.0 nm) is the trimer with three hydrogen bonds, one per water molecule. A dimer with only a single hydrogen bond is easily accommodated in a smaller cavity of 0.9-nm diameter, but is not thermodynamically stable (Fig. 1A). Linear chains of water molecules inside a narrow cylindrical pore are less stable than the water clusters in a spherical nonpolar cavity of comparable volume (28). All of the stable structures found in the spherical cavities have at least one hydrogen bond per water molecule; i.e., each water molecule is involved in two or more hydrogen bonds, a minimum number seen also for water molecules in proteins (39) and in the water chains formed inside nanotubes (12).

The stability of encapsulated water clusters depends critically on cavity size and the strength of the attractive interactions with the cavity walls. The sensitivity of filling to changes in the water–wall interactions is consistent with water filling of protein cavities upon slight increases in polarity. For instance,
during the catalytic cycle of cytochrome P450, cryocrystallography (10) indicates that the amide hydrogen of a threonine is exposed to a narrow cavity that becomes filled with a water molecule (10, 11). As indicated by our simulations using the highly accurate ASPW4 potential (33), improved water models will certainly change details of the structures and thermodynamics of water clusters in cavities but are unlikely to alter our general conclusions about water in nonpolar cavities. The spectroscopic study of water clusters trapped in cavities should be of great interest in better characterizing water in a biologically relevant environment (40). Although encapsulation into fullerences using, for instance, molecular beams or hydrothermal preparations will be challenging, other systems, in particular protein cavities, are more amenable.

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